Def: An _____ is an _____ list of numbers of the form

Ex 1

Notation: We can denote a sequence in the following ways:

(a)

(b)

(c)

(d)

Ex 2 Write the sequence from example 1 in explicit and recursive form.

Def: For the sequence, a_n, the variable n is called the _____

Note: We don't always use n for the index variable. Sometimes we use m, k, i, j instead.

Ex 3 Find an explicit formula for the sequence $\{7, 23, 39, 55, 71, 87, \dots\}$.

Def: An _____ is a sequence where the _____ between every pair of consecutive terms is _____.

Thm: Every arithmetic sequence has the form

Ex 4 Graph the sequence in 3

PQ. Which of the following are arithmetic sequences? There may be more than one correct answer.

(a) $a_n = 2n + 3$ for all $n \geqslant 0$

(b) $b_i = 2^i - 1$ for all $i \geqslant 0$

(c) $c_j = \ln(3j + 5)$ for all $j \geqslant 0$

(d) $d_k = -k$ for all $k \geqslant 0$

Ex 5 Find an explicit formula for the sequence defined by the recurrence relation $f_0 = 1$ and $f_{n+1} = \dfrac{f_n}{n+1}$ for all $n \geqslant 1$.

Ex 6 Find an explicit formula for the sequence defined by $x_0 = 3$ and $x_{n+1} = -5x_n$ for all $n \geqslant 1$.

Def: A _____ is a sequence where the _____ between every pair of consecutive terms is _____.

Thm: Every geometric sequence has the form

PQ. Which of the following defines a geometric sequence? There may be more than one correct answer.

(a) $a_n = 2 \cdot 3^n$ for all $n \geqslant 0$

(b) $b_n = 2 + 3n$ for $n \geqslant 0$

(c) $c_n = 3\cos(\pi n)$ for all $n \geqslant 0$

(d) $d_n = \frac{1}{2n+7}$ for all $n \geqslant 0$

(e) $e_n = 4 \cdot n^3$ for all $n \geqslant 0$

(f) $f_n = -2 \cdot 4^{n+1}$ for all $n \geqslant 0$

Def: The _____ $(F_n)_{n=0}^{\infty}$ is defined by the recurrence relation

Ex 7 Write out the first eight terms of the Fibonacci sequence and decide if $(F_n)_{n=0}^{\infty}$ is arithmetic, geometric, or neither.

Def: Suppose $\{a_n\}_{n=0}^{\infty}$ is a sequence.

(a) $\{a_n\}_{n=0}^{\infty}$ is _____ if there exists a number M such that _____ for all n.

(b) $\{a_n\}_{n=0}^{\infty}$ is _____ if there exists a number m such that _____ for all n.

(c) $\{a_n\}_{n=0}^{\infty}$ is _____ if it is _____

(d) $\{a_n\}_{n=0}^{\infty}$ is _____ if _____ for _____ $n \geqslant n_0$. We might also say that $\{a_n\}_{n=0}^{\infty}$ is _____

(e) $\{a_n\}_{n=0}^{\infty}$ is _____ if _____ for _____ $n \geqslant n_0$. We might also say that $\{a_n\}_{n=0}^{\infty}$ is _____

(f) $\{a_n\}_{n=0}^{\infty}$ is _____ (or _____)
if it is either _____ or
_____.

Ex 8 Recall the Fibonacci Sequence is defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$ for all $n \geqslant 1$. Is $\{F_n\}_{n=0}^{\infty}$ decreasing for all $n \geqslant 0$ or increasing for all $n \geqslant 0$ (or neither)? Is $\{F_n\}_{n=0}^{\infty}$ bounded below, above, or neither?

Ex 9 Consider the geometric sequence $\{y_n\}$ defined by $y_n = 10 \cdot \left(-\frac{1}{2}\right)^n$ for all $n \geqslant 1$. Is $\{y_n\}$ bounded? Is there an index n_0 such that $\{y_n\}$ is monotone for all $n \geqslant n_0$?

Def: (Informal) We say the sequence $\{a_n\}$ _____ to a number L if "a_n gets arbitrarily close to L" for _____ values of n. In this case, we call $\{a_n\}$ a _____ and we write

and we call L the _____ of $\{a_n\}$.

Def: (Formal) We say the sequence $\{a_n\}$ _____ to the limit L if

Q. What does it look like for a sequence a_n to converge to L?

Q. What does it look like for a sequence a_n to converge to something other than L?

Here's a second way of thinking about convergence. $\lim_{n\to\infty} a_n = L$ if we can have the following conversation:

Ex 10 Suppose $x_n = \left(-\frac{1}{7}\right)^n$ for all $n \geqslant 1$. Let $\varepsilon = 0.0005$. Find an explicit integer n_0 with the property that $|x_n - 0| < \varepsilon$ for all $n \geqslant n_0$.

Ex 11 How do you convince someone $\lim_{n\to\infty} \frac{2n^2}{1+n^2} = 2$?

Ex 12 Compute $\lim_{n\to\infty} r^n$ for any (constant) r.

Def: If a sequence $\{a_n\}$ does not converge to any limit, we say $\{a_n\}$ _____ or

_____

Note A sequence can diverge in the following ways:

(a)

(b)

(c)

Def: A sequence $\{a_n\}$ _____ if

Def: A sequence $\{a_n\}$ _____ if

Thm: (Squeeze Theorem for Sequences) Suppose _____ for all $n \geqslant n_0$ and suppose that

_____. Then _____

Ex 13 Let $b_n = \dfrac{\sin(2^n)}{2^n}$. Compute $\lim_{n\to\infty} b_n$.

Ex 14 Compute $\lim_{n\to\infty} \dfrac{n!}{n^n}$. But guess first!

PQ.
(a) 0
(b) 1
(c) ∞
(d) $-\infty$
(e) e
(f) π
(g) The limit does not exist

Thm: (Algebraic Limit Laws) If $\lim_{n\to\infty} a_n = A$, $\lim_{n\to\infty} b_n = B$, and c is a constant, then:

(a) $\lim_{n\to\infty} c = c$

(b) $\lim_{n\to\infty} ca_n = c \cdot \lim_{n\to\infty} a_n = cA$

(c) $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n = A \pm B$

(d) $\lim_{n\to\infty} (a_n \cdot b_n) = \left(\lim_{n\to\infty} a_n\right) \cdot \left(\lim_{n\to\infty} b_n\right) = A \cdot B$

(e) $\lim_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n} = \dfrac{A}{B}$, provided $b_n \neq 0$ for all n and $B \neq 0$.

Ex 15 Determine whether the sequence $\left\{\dfrac{3n^4 - 7n^2 + 5}{6 - 4n^4}\right\}$ converges. If it converges, then compute the limit.

Ex 16 Why can't we use the Algebraic Limit Laws to directly compute $\lim_{n\to\infty}(\sqrt{n+1}-\sqrt{n})$?

Thm: (Continuous Functions Defined on Convergent Sequences) Suppose that $\{a_n\}$ and L are in the _____ of a function $f(x)$. Suppose that $f(x)$ is _____ at L and that _____. Then

Ex 17 Compute $\lim_{n\to\infty}(\ln(2+3n)-\ln(n))$.

Q. How are boundedness and monotonicity related to convergence?

Thm: (Monotone Convergence) If a sequence $\{a_n\}$ is _____ for all $n \geqslant n_0$, then

Ex 18 $\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}} = ?$